Part I: Dimensional Analysis (aka Factor-Label or Unit Cancellation Method)

Sample Problem 1
A large, coal-fired electric power plant produces 12 million kilowatt-hours of electricity each day. Assume that an input of 10,000 BTUs of heat is required to produce an output of 1 kWh of electricity.

(a) Calculate the number of BTUs of heat needed to generate the electricity produced by the power plant each day.

(b) Calculate the number of pounds of coal consumed by the power plant each day, assuming that one pound of coal yields 5,000 BTUs of heat.

(c) Calculate the number of pounds of sulfur released by the power plant each day, assuming that the coal contains one percent sulfur by weight.

Where to start?

Step 1: Ask yourself, “What unit of measure do I need for my answer?”

Read part (a) again. You need to answer in units of “BTUs per day”. The word “per” is code for “divided by” or “in the denominator”. Next, translate the words into math. So, “BTUs per day” becomes \( \frac{\text{BTU}}{\text{day}} \).

Consider this your end goal; it is how you must express your final answer.

Step 2: Am I given any numbers that are paired with the units I seek?

Not exactly! That would be WAY too easy. But you can identify a “bridge” of sorts that connects "BTU" and "day". You are given “power plant produces 12 million kilowatt-hours (12,000,000 kWh) of electricity each day” AND “10,000 BTU” of heat is required to produce an output of 1 kWh of electricity”. From this we can write two equivalent sets of conversion factors for each of these tidbits of valuable information.

\[
\frac{12,000,000 \text{ kWh}}{1 \text{ day}} \quad \text{or} \quad \frac{1 \text{ day}}{12,000,000 \text{ kWh}} \quad \text{as well as} \quad \frac{1 \text{ kWh}}{10,000 \text{ BTU}} \quad \text{or} \quad \frac{10,000 \text{ BTU}}{1 \text{ kWh}}
\]

These are called conversion factors and they are your friends! Prepositions are also your friends since they link information together as in “pieces of candy” and “at 15 °F”. Your objective is to string the conversion factors together (keeping the numbers with the units) so that the unwanted units cancel out leaving you with the desired units for your answer. In our example the desired units are \( \frac{\text{BTU}}{\text{day}} \). The numerator and denominator of a conversion factor represent the same quantity, such as \( \frac{12 \text{ inches}}{1 \text{ foot}} \) or \( \frac{1 \text{ foot}}{12 \text{ inches}} \) thus we are simply multiplying by “1”.

Step 3: Choose the conversion factors AND structure them so that you achieve your “end goal”.

Remember we were asked to solve for “BTUs per day” which in “math speak” looks like units of \( \frac{\text{BTU}}{\text{day}} \).

It there is a “trick” to this process, it involves choosing conversion factors and arranging them so that the unwanted units cancel out leaving only the desired unit(s) on your final answer! For our example, you want BTU in the numerator of your final answer along with days in the denominator. So, you need to get rid of or cancel the kWh. How?

\[
\frac{12,000,000 \text{ kWh}}{1 \text{ day}} \quad \text{and} \quad \frac{10,000 \text{ BTU}}{1 \text{ kWh}}
\]

make a nice pair since they result in the desired units of \( \frac{\text{BTU}}{\text{day}} \). Yippee!

Step 4: Solve it! Let’s revisit our sample problem:

When you have cancelled out the units you don’t want and are left only with the units you do want, then you know it’s time to multiply all the numerators (top numbers) together, and divide by EACH of the bottom numbers! When setting up a string of conversion factors, it is often helpful to start at the end (units of the desired answer) and work backwards as explained and demonstrated on the screencast.

**Sample Problem 1**

A large, coal-fired electric power plant produces 12 million kilowatt-hours of electricity each day. Assume that an input of 10,000 BTUs of heat is required to produce an output of 1 kWh of electricity.

(a) **Calculate** the number of BTUs of heat needed to generate the electricity produced by the power plant each day.

(b) **Calculate** the number of pounds of coal consumed by the power plant each day, assuming that one pound of coal yields 5,000 BTUs of heat. (Pick up where you left off! Let the problem lead you!)

(c) **Calculate** the number of pounds of sulfur released by the power plant each day, assuming that the coal contains one percent sulfur by weight.
Sample Problem 2

The environmental impact of washing a load of dirty dishes in an electric dishwasher differs from that of washing them in a sink by hand. A comparison of the two methods may allow consumers to spend their money more wisely. Use the assumptions in the table below to perform the calculations that follow.

<table>
<thead>
<tr>
<th>All the dishes fit into a single load.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The water entering both the water heater and the sink is at 50 °F.</td>
</tr>
<tr>
<td>The water heater and the dishwasher are both 100% efficient.</td>
</tr>
<tr>
<td>Washing the dishes by hand requires 20 gallons of water heated to 110 °F.</td>
</tr>
<tr>
<td>In one complete cycle, the electric dishwasher uses 10 gallons of water heated to 140 °F and the dishwasher also uses 0.500 kilowatt-hour of electrical energy for its mechanical operations.</td>
</tr>
</tbody>
</table>

(a) A British thermal unit or BTU is the amount of energy needed to raise the temperature of one pound of water 1.0 °F. Additionally, one gallon of water has a mass of 8.0 pounds. **Calculate** the total energy used to wash a load of dishes using the dishwasher.

(b) One kilowatt-hour is equivalent to 3,400 BTUs and the cost of electricity is $0.11 per kWh. **Calculate** the energy in BTU used to heat the water for washing a load of dishes by hand.

(c) **Calculate** the cost of electricity for each dishwashing method given that one kilowatt-hour is equivalent to 3,400 BTUs and the cost of electricity is $0.11 per kWh.